Relationship between Corporate Yield Spread and Treasury Yield

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Merton's Model

- The firm asset follows

$$\mathrm{d}A_t = rA_t\mathrm{d}t + \sigma A_t\mathrm{d}B_t^{\mathbb{Q}} \tag{1}$$

- The firm finances through equity and debt. Assume firm issues a zero coupon bond with face value F and maturity T.
- If $A_T < F$, then the firm defaults and transfer all of its assets to debtholders.
- When r increases, the drift of the firm's asset increases, **lowering the default probability** and, therefore, the yield spread.

Extensions

- Black and Cox (1992): Early bankruptcy. A firm defaults whenever its value falls below a specified default trigger K. Even under this assumption, rising interest rates are still expected to increase firm value, thus lowering the probability of default.
- Longstaff and Schwartz (1995): Stochastic interest rate. Since CIR model is strictly increasing with r_0, then the result is still negative relationship.
- Duffie and Lando (2001): Imperfect information. Previous structural models predict the credit spread will be zero when maturity is close to 0. But in market, those bonds close to maturity may have large credit spread.
- Optimal Capital Structure and Default: Leland (1994), Leland and Toft (1996). See next slides.

Leland (1994)

- At $t = 0$, the owners of a debt-free firm decide to issue debt to optimize their equity value.
- Two control parameters: (1) K the default trigger; (2) D_0 the size of the debt.
- Other Parameters: (1) $\tau \in [0, 1]$: Tax rate. (2) $\alpha \in [0, 1)$: The fraction of asset value lost at the time of bankruptcy due to frictions.
- The debt is a **perpetual bond** that pays a constant coupon rate C every unit of time.
- Firm Asset:

$$\mathrm{d}A_t = (r - \delta)A_t\mathrm{d}t + \sigma\mathrm{d}B_t^{\mathbb{Q}} \tag{2}$$

Leland (1994)

- Default is triggered when $A_t \leq K$:

$$\tau_B := \inf\{t : A_t \leq K\} \tag{3}$$

- At time $t = \tau_B$, the debt value is $D_{\tau_B} = (1-\alpha)K$.
- Debt Value:

$$D_0 = \underbrace{\mathbb{E}^{\mathbb{Q}}[e^{-r\tau_B}(1-\alpha)\mathbf{1}_{\{0\leq\tau_B<\infty\}}]}_{\text{Expected Liquidation Value at Default}} + \underbrace{\mathbb{E}^{\mathbb{Q}}\left[\int_0^{\tau_B} Ce^{-rt}\mathrm{d}t\right]}_{\text{Coupons}} \tag{4}$$

- Firm Asset:

$$V_0 = A_0 + \underbrace{\mathbb{E}^{\mathbb{Q}}\left[\int_0^{\tau_B} \tau Ce^{-rt}\mathrm{d}t\right]}_{\text{Tax Shield}} - \underbrace{\mathbb{E}^{\mathbb{Q}}[e^{-r\tau_B}\alpha K\mathbf{1}_{\{0\leq\tau_B<\infty\}}]}_{\text{Expect Loss on Liquidation}} \tag{5}$$

Leland (1994)

- Using Laplace Transform, we can get closed form solution to debt value and firm asset value.
- Debt:

$$D_0 = (1-\alpha)K\left(\frac{A_0}{K}\right)^{-\gamma} + \frac{C}{r}\left[1-\left(\frac{A_0}{K}\right)^{-\gamma}\right] \qquad (6)$$

- Firm Asset:

$$v_0 = A_0 + \frac{\tau C}{r}\left[1-\left(\frac{A_0}{K}\right)^{-\gamma}\right] - \alpha K\left(\frac{A_0}{K}\right)^{-\gamma} \qquad (7)$$

where

$$\mu = \frac{1}{\sigma}\left(\frac{1}{2}\sigma^2 - r + \delta\right) \geq 0, \quad \gamma = \frac{\sqrt{\mu^2 + 2r} - \mu}{\sigma} > 0 \quad (8)$$

Leland (1994)

- Equity Value:

$$E_0 = v_0 - D_0 = A_0 - \frac{(1-\tau)C}{r}\left[1 - \left(\frac{A_0}{K}\right)^{-\gamma}\right] - K\left(\frac{A_0}{K}\right)^{-\gamma} \tag{9}$$

- Smooth Pasting Condition: we must make sure that given C, $E_0 \geq 0$ for all $A_0 \geq K$. Hence $\left.\frac{\mathrm{d}E_0}{\mathrm{d}A_0}\right|_{A_0=K} = 0$.
- Endogeneous Default Barrier:

$$K^* = \frac{(1-\tau)C}{r}\frac{\gamma}{1+\gamma} \tag{10}$$

- C^* maximizes v_0:

$$C^* = A_0\frac{r(1+\gamma)}{(1-\tau)\gamma}\left(1+\gamma+\frac{\alpha\gamma(1-\tau)}{\tau}\right)^{-\frac{1}{\gamma}} \tag{11}$$

Leland (1994): Unprotected Debt

- Yield Spread is defined as:

$$R = \frac{C}{D} - r \tag{12}$$

- Result: Yield spread is **decreasing** with interest rate.
- Why? Still need a better explanation.
- The default barrier is unconstrained. It can be less than the principle value of the bonds, $P = D_0$.

Leland (1994): Protected Debt

- Now suppose that the debt is proctected, i.e. the firm value must be higher than principal value of bonds $P = D_0$. Set $K = D_0$.
- Then debt is given by the equation

$$D_0 = (1-\alpha) D_0 \left(\frac{A_0}{D_0}\right)^{-\gamma} + \frac{C}{r}\left[1 - \left(\frac{A_0}{D_0}\right)^{-\gamma}\right] \tag{13}$$

- The model predicts that yield spreads will *increase* as interest rates rise.

Leland and Toft(1996)

- Finite maturity bonds.
- For long-term debt, the endogeneous default barrier K is typically **less** than principle of bonds, i.e. $K < P$. Hence the firm may continue to operate despite having negative net worth.
- But when $T \to 0$, then $K \to P/(1-\alpha)$, it exceeds P. Hence for short-term debt, it is always **protected**. For short term bonds, bankruptcy will occur despite net worth being positive!
- For newly issued debt, the risk free rate can **increase** the short term bonds' credit spread.
- But for long term debt, risk free rate will still decrease the credit spread.

Inference

- Leland (1994) claims that **protected** bonds will have increasing relationship between spread and treasury yield.
- But in reality, such a covenant is not common. I consider "protected" to mean "high credit rating".
- Leland and Toft (1996) states that **short** maturity bond may have increasing relationship.
- Short maturity bond alone may not enough. We have to find more specific group of bonds that may have positive relationship.
- The positive relationship can be interpreted as: when the interest rate rises, the firm will be more difficult to get money from debt.
- This can also be viewed as liquidity problem.

Regression

- Data: We constructed our bond panel data from TRACE and the Mergent FISD.
- We exclude all the bonds with floating rate coupons and embedding options.
- Moreover, we exclude all the bonds whose issuer is in financial industry.
- Consider the following regression equation:

$$\begin{aligned} \mathrm{CS}_{i,t} = \beta_0 + \beta_1 Y_t + \beta_2 \mathrm{Term}_t + \beta_3 \mathrm{TMT}_{i,t} + \beta_4 \mathrm{Age}_{i,t} \\ + \beta_5 \mathrm{Size}_i + \beta_6 \mathrm{Coupon}_i + \beta_7 \mathrm{Vol}_{i,t} \end{aligned} \tag{14}$$

Empirical Result

	Investment Grade Bonds				Speculative Grade Bonds			
	Low Zerodays		High Zerodays		Low Zerodays		High Zerodays	
Intercept	1.23	1.43	1.83	1.53	7.95	-6.23	7.26	-1.99
	[53.04]	[9.40]	[34.47]	[17.66]	[16.90]	[-2.52]	[24.08]	[-1.00]
DGS1	-0.036	$\boxed{\mathbf{0.043}}$	-0.14	-0.067	-1.27	$\boxed{-0.050}$	-0.90	-0.39
	[-5.10]	[**7.30**]	[-9.77]	[-3.66]	[-8.19]	[-0.47]	[-8.94]	[-4.32]
Term	-0.067	0.066	-0.24	-0.16	-1.66	0.35	-1.17	-0.41
	[-3.80]	[4.72]	[-7.65]	[-3.20]	[-6.13]	[1.71]	[-6.43]	[-2.31]
TMT		0.022		0.019		-0.0200		0.25
		[26.41]		[5.73]		[-1.38]		[3.83]
Age		-0.055		-0.031		0.0003		0.060
		[-17.49]		[-4.25]		[0.011]		[1.95]
Size		-0.15		-0.16		0.017		-0.26
		[-14.51]		[-12.66]		[0.096]		[-2.38]
Coupon		0.29		0.32		0.76		0.83
		[41.12]		[5.93]		[10.84]		[12.71]
Volatility		2.32		2.13		13.34		8.16
		[27.41]		[31.89]		[13.65]		[10.89]
Obs	207,312		304,226		52,384		37,770	
Adj-R^2	0.001	0.476	0.004	0.119	0.011	0.294	0.01	0.282

Liquidity effect on Yield Spread

- Huang and Huang (2012): Credit risk only accounts for small fraction of yield spread. The fraction is lower for bonds with short maturity. And the fraction is higher for high yield bonds.
- Most of paper just measure liquidity risk and use it as an explanatory variable in regression.
- Ericsson and Renault (2006): They propose a structural model incorporating liquidity risk in bonds. λ_t: the instantaneous probability of being forced to sell. This model is more complex than previous model.
- He and Xiong (2012): They extend Leland and Toft (1996) with illiquidity. They also model the illiquidity as shock that investor is forced to sell his bond at a fractional cost.
- Chen et al. (2018): They model illiquidity as individual will bear a fixed holding cost.

He and Xiong (2012)

- In addition to Leland and Toft (1996), each bond investor will face a liquidity shock which arrives to a Poisson occurence with intensity ξ.
- Upon the arrival of the shock, bond investor has to exit by selling his bond holding at a fractional cost of k.
- Result: The formula is the same as Leland and Toft (1996) but with discount rate $r + \xi k$.
- Then it has similar result as in Leland and Toft (1996), that is credit spread is decreasing with interest rate.

He and Xiong (2012)

- But He and Xiong (2012) has different definition of spread.
- In Leland and Toft(1996), spread is defined as

$$\Delta = \frac{c}{d} - r \tag{15}$$

- He and Xiong (2012) defines yield y as solution to the following equation:

$$d(V_t, T) = \frac{c}{y}(1 - e^{-yT}) + pe^{-yT} \tag{16}$$

- When $d = p$, the solution is exactly the same.
- Actually, $d - p$ capture equity holders' rollover gain/loss from paying off maturing bonds by issuing new bonds at the market price.

He and Xiong (2012)

- What if r and ξ has correlation/functional relationship?
- If r and ξ has a negative relationship.
- The economic interpretation is that when the interest rate is high, then firm are easier to issue debt since market. (But this interpretation may not be what ξ implies).
- Other interpretation?
- Or r and k?

Liquidity Structural Model

- Can we find a least complex model that capturing the liquidity risk and credit risk?
- Start from Leland and Toft (1996)?